**SOME PRIME LABELING OF GRAPH**

Dissertation submitted to **H.H.THE RAJAH’S COLLEGE (Autonomous B+)**

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Submitted by

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**CERTIFICATE**

This is to certify that the dissertation entitled **“ SOME PRIME LABELING OF GRAPH ”** submitted in partial fulfillment of the requirements for the award of the Degree of **MASTER OF SCIENCE** In Mathematics, H.H. THE RAJAH’S COLLEGE (Autonomous B+), Pudukkottai, Bharathidasan University, Tiruchirappalli, is a record of this research work done by **R. SARANRAJ (Reg. No. 22PMT 4116)** under my supervision and guidance during the academic period (2023 - 2024).

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**DECLARATION**

I hereby declare that the dissertation entitled, **“ SOME PRIME LABELING OF GRAPH ”** submitted to H. H. The Rajah’s college (Autonomous B+), Pudukkottai, Bharathidasan University, Tiruchirappalli in partial fulfillment of the requirements for the award of the Degree of **MASTER OF SCIENCE** in Mathematics a record of work done by me under the supervision and guidance. **Mrs. K. NACHAMMAL, M.Sc., M.Phil., PGDCA.,** Assistant Professor, Department of Mathematics, H.H. The Rajah’s College (Autonomous B+), Pudukkottai and this dissertation has not formed the basis for the award of any Degree/ Diploma/ Fellowship or other similar titles to any candidate of any University.

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**CHAPTER I**

**INTRODUCTION**

Throughout this project, we consider only finite simple undirected graph. The graph G has vertex set and edge set .

The labeling of a graph G is an assigning of integers either to the vertices or edges or both subject to certain conditions.

The notion of a prime labeling was introduced by Roger Entringer and was discussed in a paper by Tout.A. Many researchers have studied prime graph for example in Fu.H. have proved that the path on n vertices is a prime graph.

In Dretsky.T have proved that the on n vertices is a prime graph. In Lee.S have proved that the wheel is a prime graph if and only if n is even. In Vaidya.k have proved the prime labeling for some cycle related graphs.

In this project, we study some prime labeling of graph.

**CHAPTER II**

**CHAPTER II**

**PRELIMINARIES**

**Definition 2.1**

A graph G consist of a pair (V(G),E(G)) where V(G) is a non empty finite set whose elements are called points or vertices and E(G) is another set of unordered pairs of distinct elements of V(G). The elements of E(G) are called edges of graph.

**Example 2.2**

**Fig 2.1: Graph (G)**

**Definition 2.3**

A graph g is said to be a subgraph of a graph G if all the vertices and all the edges of g are in G, and each edge of g has the same end vertices in g as in G.

**Example 2.4**

**f**

**5**

**a**

**1**

**h**

**a**

**1**

**c**

**b**

**4**

**g**

**c**

**b**

**4**

**2**

**e**

**2**

**i**

**e**

**d**

**j**

**d**

**6**

**3**

**3**

**(b)**

**(a)**

**Fig 2.2: Graph (a) and one of its subgraph (b)**

**Definition 2.5**

Let G be a graph. A walk is defined as a finite alternating sequence of vertices and edges. Beginning and ending with vertices, such that each edge is incident with the vertices preceding and following it.

For example,

Fig 2.3 **a 5 b 2 c 3 d 1 e** is a walk.

**e**

**1**

**Example 2.6**

**d**

**c**

**4**

**3**

**a**

**b**

**5**

**2**

**Fig 2.3: Graph (G)**

**Definition 2.7**

A walk to begin and end at the same vertex, such a walk is called a closed walk. A walk is not closed is called an open walk.

**Definition 2.8**

An open walk in which no vertex appears more than once is called a path.

For example,

Fig 2.4 **a 1 b 3 e 5 d 8 c** is a path and **a 1 b 3 e 5 d 6 b 2 c** is not a path.

**Definition 2.9**

A closed walk in which no vertex (except the initial and the final vertex) appears more than once is called a circuit or cycle. That is, a cycle is a closed nonintersecting walk.

For example,

Fig 2.4  **a 1 b 2 c 8 d 5 e 7 a** is a cycle.

**Example 2.10**

**Fig 2.4: Graph (G)**

**8**

**7**

**6**

**5**

**4**

**3**

**2**

**1**

**e**

**d**

**c**

**b**

**a**

**Definition 2.11**

The number of edges incident on a vertex  **,** with self-loops counted twice, is called the degree, of vertex .

For example,

Fig 2.4 = = = = 3 and = 4

**Definition 2.12**

The sum of the degree of all vertices in G is twice the number of edges in G. That is,

**Definition 2.13**

A vertex of degree one is called a pendent vertex.

For example,

Fig 2.5 d(A) = d(C) = d(D) = d(E) = 1. Hence A,C,D,E is a pendent vertex.

**Example 2.14**

**C**

**B**

**A**

**D**

**Fig 2.5: Graph (G)**

**E**

**Definition 2.15**

A graph in which all vertices are of equal degree is called a regular graph or simply a regular. If every vertex in a graph G has the same degree r, then the graph G is called a regular graph of degree r, or r-regular graph.

**Example 2.16**

**B**

**A**

**C**

**D**

**Fig 2.6: r-regular graph**

**Definition 2.17**

A graph G is said to be connected if there is at least one path between every pair of vertices in G. Otherwise, G is disconnected.

**Example 2.18**

**Fig 2.7: Connected graph**

**Definition 2.19**

A graph that has no self-loops and parallel edges is called a simple graph.

**Example 2.20**

**A**

**B**

**C**

**Fig 2.8: Simple graph**

**Definition 2.21**

An undirected graph is graph. That is, a set of vertices are connected together, where all the edges are bidirectional.

**Example 2.22**

**Fig 2.9: Undirected graph**

**Definition 2.23**

A bipartite graph is one whose vertex set can be partitioned into two subsets X and Y, So that each edge has one end in X and one end in Y; Such a partition (X,Y) is called a bipartition of the graph.

**Example 2.24**

**Fig 2.10: Bipartite graph**

**Definition 2.25**

A complete bipartite graph is a simple bipartite graph with bipartition (X,Y) in which each vertex of X is joined to each vertex of Y; if and , such a graph is denoted by .

**Example 2.26**

**Fig 2.11: Complete Bipartite graph**

**Definition 2.27**

A simple graph in which there exists an edge between every pair of vertices is called a complete graph.

**Example 2.28**

**B**

**A**

**C**

**D**

**Fig 2.12: Complete graph**

**Definition 2.29**

A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions.

**Definition 2.30**

A graph in which each vertex is assigned unique name or label that graph is called the labeled graph.

**Example 2.31**

**Fig 2.13: Labeled graph**

**CHAPTER III**

**CHAPTER III**

**PRIME LABELING OF CERTAIN GRAPHS**

**Definition 3.1**

Let be a graph with p-vertices and q-edges. A bijection is called a prime labeling if for each edge e =, such that A graph which admits prime labeling is called a prime graph.

**Definition 3.2**

The Friendship graph is a set on n triangles having a common central vertex.

**Example 3.3**

**Fig 3.1: Friendship graph**

**Theorem 3.4**

The Friendship graph is a prime graph.

**Proof:**

Let = with as the centre vertex.

Let = U

Define a labeling f by

→ as follows

Let for 1 ≤ i ≤ 2n+1

Now,

for 2 ≤ i ≤ 2n+1

for 1 ≤ i ≤ n

Then f admits prime labeling.

Hence is a prime graph.

**Example 3.5**

**2**

**13**

**12**

**11**

**3**

**4**

**10**

**1**

**9**

**5**

**6**

**7**

**8**

**Fig 3.2: Prime labeling of friendship graph**

**Definition 3.6**

A fan graph obtained by joining all vertices of , n ≥ 2 is a path to a further vertex called the center.

Thus contains n+1 vertices say and (2n-1) edges say , 1 ≤ i ≤ n and , 1 ≤ i ≤ n-1.

**Example 3.7**

**c**

**Fig 3.3: Fan graph**

**Theorem 3.8**

The Fan graph is a prime graph.

**Proof:**

Let G = be a fan graph.

Let

Let U

Define a labeling f by

→ as follows

Let

for 1 ≤ i ≤ n

Now,

for 1 ≤ i ≤ n

for 1 ≤ i ≤ n-1

Then f admits prime labeling.

Hence G is a prime graph.

**Example 3.9**

**2**

**6**

**5**

**4**

**3**

**1**

**Fig 3.4: Prime labeling of fan graph**

**Definition 3.10**

The Franklin graph is a 3-regular graph with 12 vertices and 18 edges.

**Example 3.11**

**Fig 3.5: Franklin graph (FG)**

**Theorem 3.12**

The Franklin graph FG is a prime graph.

**Proof:**

Let FG be the Franklin graph with 12 vertices and 18 edges.

Let

Let U U U U

and

Define a labeling f by

→ as follows

Let for 1 ≤ i ≤ 12

Now,

for 1 ≤ i ≤ 11

for 1 ≤ i ≤ 2

for 3 ≤ i ≤ 4

for 5 ≤ i ≤ 6

Then f admits prime labeling.

Hence FG is a prime graph.

**Example 3.13**

**12**

**1**

**11**

**2**

**10**

**3**

**9**

**4**

**8**

**5**

**7**

**6**

**Fig 3.6: Prime labeling of Franklin graph**

**Definition 3.14**

The Heawood graph is an undirected graph with 14 vertices and 21 edges. Heawood graph is a 3-regular graph.

**Example 3.15**

**Fig 3.7: Heawood graph**

**Theorem 3.16**

The Heawood graph is a prime graph.

**Proof:**

Let G be the heawood graph with 14 vertices and 21 edges.

Let

Let U U U U

and

Define a labeling f by

→ as follows

Let for i = 2,3,4,6,7,8,9,10,11,12,13,14

Now,

for 1 ≤ i ≤ 13

for i = 1,3,5,7,9

Then f admits prime labeling.

Hence G is a prime graph.

**Example 3.17**

**Fig 3.8: Prime labeling of Heawood graph**

**4**

**14**

**13**

**12**

**11**

**10**

**9**

**8**

**7**

**6**

**1**

**3**

**2**

**5**

**CHAPTER IV**

**CHAPTER IV**

**PRIME LABELING OF DIFFERENT GRAPHS**

**Definition 4.1**

A simple graph of ‘n’ vertices (n ≥ 3) and n edges forming a cycle of length ‘n’ is called as a cycle graph. In a cycle graph, all the vertices are of degree is 2. By adding the path, the new vertices of , and the new graph G is denoted by .

**Theorem 4.2**

The cycle and path graph are a prime graph. Then the graph is a prime labeling of the graph.

**Proof:**

Let G be the graph obtained by joining cycle and a path .

To prove the graph admits to prime labeling of the graph.

Let be the vertices of cycle .

Let be the vertices of path .

Define a labeling,

Let for 1 ≤ i ≤ n

Assume

Let for 1 ≤ i ≤ (m-1)

Now,

for 1 ≤ i ≤ n

for 1 ≤ i ≤ (m-1)

Then f admits prime labeling.

Hence is a prime graph.

**Example 4.3**

**2**

**1**

**7**

**8**

**9**

**10**

**6**

**3**

**5**

**4**

**Fig 4.1: Prime labeling of graph**

**Definition 4.4**

The crown graph on 2n vertices is an undirected graph with two set of vertices and and with on edge from to whenever i≠j. By adding the path, the new vertices of , and the new graph G is denoted by crown .

**Theorem 4.5**

The crown and path graph are a prime graph. Then the graph is a prime labeling of the graph.

**Proof:**

Let G be the graph obtained by joining crown by a path .

To prove the graph admits to prime labeling of the graph.

Let be the vertices of crown .

Let be the vertices of crown .

Let be the vertices of path .

Define a labeling,

Let for 1 ≤ i ≤ n

for 1 ≤ i ≤ n

Assume

Let for 1 ≤ i ≤ (m-1)

Now,

for 1 ≤ i ≤ n

for 1 ≤ i ≤ n

for 1 ≤ i ≤ (m-1)

Then f admits prime labeling.

Hence is a prime graph.

**Example 4.6**

**2**

**1**

**5**

**3**

**9**

**8**

**7**

**6**

**4**

**Fig 4.2: Prime labeling of graph**

**Definition 4.7**

The Friendship graph is a graph which consists of n-triangles with a common vertex. If and by adding the path, the new vertices of , and the new graph G is denoted by .

**Theorem 4.8**

The friendship and path graph are a prime graph. Then the graph is a prime labeling of the graph.

**Proof:**

Let G be the graph obtained by joining friendship by a path .

To prove the graph admits to prime labeling of the graph.

Let be the vertices of friendship .

Let be the vertices of path .

Define a labeling,

Let for 1 ≤ i ≤ (2n+1)

Assume

Let for 1 ≤ i ≤ (m-1)

Now,

for 2 ≤ i ≤ (2n+1)

for 1 ≤ i ≤ n

for 1 ≤ i ≤ (m-1)

Then f admits prime labeling.

Hence is a prime graph.

**Example 4.9**

**3**

**2**

**4**

**8**

**9**

**10**

**11**

**12**

**1**

**5**

**7**

**6**

**Fig 4.3: Prime labeling of graph**

**Definition 4.10**

The star graph is special type of graph in which n-1 vertices have degree 1 and single vertex have n-1 degree. This look like n-1 vertex is connected to central vertex. A star graph with total n vertex is termed as . By adding the path, the new vertices of , and the new graph G is denoted by .

**Theorem 4.11**

The star and path graph are a prime graph. Then the graph is a prime labeling of the graph.

**Proof:**

Let G be the graph obtained by joining star by a path .

To prove the graph admits to prime labeling of the graph.

Let be the vertices of star .

Let be the vertices of path .

Define a labeling,

Let

for 1 ≤ i ≤ n

Assume

Let for 1 ≤ i ≤ (m-1)

Now,

for 1 ≤ i ≤ n

for 1 ≤ i ≤ (m-1)

Then f admits prime labeling.

Hence is a prime graph.

**Example 4.12**

**3**

**4**

**2**

**1**

**13**

**12**

**11**

**10**

**9**

**5**

**6**

**8**

**7**

**Fig 4.4: Prime labeling of graph**

**Definition 4.13**

The Gear graph also known as a bipartite wheel graph is a wheel graph with a vertex added between each pair of adjacent vertices of the outer cycle, further vertex is called center. Gear graph has 2n+1 vertices and 3n edges. By adding the path, the new vertices of , and the new graph G is denoted by .

**Theorem 4.14**

The Gear and path graph are a prime graph. Then the graph is a prime labeling of the graph.

**Proof:**

Let G be the graph obtained by joining Gear by a path .

To prove the graph admits to prime labeling of the graph.

Let and be the vertices of Gear .

Let be the vertices of path .

Define a labeling,

Let

for 1 ≤ i ≤ n

for 1 ≤ i ≤ n

Assume

Let for 1 ≤ i ≤ (m-1)

Now,

for 1 ≤ i ≤ n

for 1 ≤ i ≤ n

for 1 ≤ i ≤ n

for 1 ≤ i ≤ (m-1)

Then f admits prime labeling.

Hence is a prime graph.

**Example 4.15**

**4**

**5**

**3**

**6**

**2**

**1**

**17**

**16**

**15**

**14**

**13**

**7**

**8**

**12**

**9**

**11**

**10**

**Fig 4.5: Prime labeling of graph**

**CHAPTER V**

**CONCLUSION**

Prime labeling has been studied for the five decades. A huge number of research articles published in the area of graph theory and discrete mathematics.

In the first chapter, we discuss, “Introduction to the labeling of a graph” , “Introduction to the prime labeling of a graph”.

In the second chapter, we present some basic definitions which are needed to the subsequent chapters.

In the third chapter, we study the prime labeling of certain graphs for, friendship graph , fan graph , franklin graph FG , heawood graph.

In the fourth chapter, we study the prime labeling of different graphs for, , , crown , and in necessary conditions, In future work for some connected graphs.

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**REFERENCE**

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